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Comment on 4D Lorentz invariance violations in the brane-world

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ABSTRACT: The brane-world scenario offers the possibility for signals to travel outside our visible universe and reenter it. We find the condition for a signal emitted from the brane to return to the brane. We study the propagation of such signals and show that, as seen by a 4D observer, these signals arrive earlier than light traveling along the brane. We also study the horizon problem and find that, while the bulk signals can travel far enough to homogenize the visible universe, it is unlikely that they have a significant effect since they are redshifted in the gravitational field of the bulk black hole.

KEYWORDS: Extra Large Dimensions, Physics of the Early Universe.

Contents

1. Introduction

The brane-world scenario offers possible solutions to the hierarchy problem [1] and it predicts new effects, like modifications of Newton's force law at short distances [2], modifications of the cosmological evolution at early times [3]–[8] and the existence of "faster-than-light" gravitational signals [9]–[13]. The last effect is due to the fact that, while the standard model fields (namely the photons) are confined to live only inside the brane world-volume, gravity can see the whole higher-dimensional space-time. Consequently, a "shorter" path that leaves the brane, propagates through the bulk, and re-enters the brane later may exist for gravitons.

This effect was pointed out in ref. [14], where the authors argued that all points in the visible universe can be connected in an arbitrarily short amount of time. The effect was also used in ref. [15] to give a solution to the horizon problem without using inflation. In ref. [16] the author argues that however, closed time-like curves do not form, and therefore causality is maintained. The causal structure and the horizon problem in the absence of inflation are also studied in ref. [17], and the holographic interpretation of the Lorentz invariance violations is given in ref. [18].

Since the 5D space-time is usually chosen to be AdS-Schwarzschild (AdSS) or AdS-Reissner-Nördstrom (AdS-RN) in order to localize gravity and reproduce Newton's law in the brane, the 5D metric has the general form of ref. [19] :

$$ds^2 = -f(r)dt^2 + r^2 d\Sigma_k^2 + \frac{dr^2}{f(r)} \,. \tag{1.1}$$

The 4D universe is seen as a 3-brane moving through this background. Since there is no preferred orientation of the brane, there is no reason to expect Lorentz invariance on the brane. The problem is addressed in ref. [20].The proper time of a 4D observer (as a function of the 5th dimension) changes differently than the scale factor of the space coordinates. Consequently, the effective speed of light changes as a function of the position along the 5th dimension, and it is possible that, in a given coordinate time interval, a gravitational wave signal traveling outside our visible universe will move over larger distances than light wave signals traveling inside the visible universe. The same effect can affect the standard model particles if they are assumed to be localized modes of bulk fields, the particles being allowed to "tunnel" into the bulk (see ref. [9, 21]).

In ref. [12] the authors use the more general AdS-RN bulk and they consider only flat 3-dimensional spatial slices of the visible universe. Additionally they impose the existence of an event horizon in the bulk, and these constraints require the existence of exotic matter on the brane.

In the present work we discuss the conditions under which the geodesics will return to the brane and we estimate the relative advance of the gravitational signal with respect to the light signal traveling along the brane. We will consider more generic situations, without constraining the curvature to be zero, but assume that the cosmic scale factor of the universe is large at present. The static and the expanding cases will be treated separately since we will see that we cannot take the limit $H \to 0$ in the expanding case. The analysis of the 5D null geodesics is essentially that of ref. [13], (see ref. [17] for a more complete analysis including time-like and space-like geodesics) but in addition we explicitly impose the condition that the geodesics return to the brane. Only then such apparent Lorentz invariance violations can be detected by a brane observer. We also use the analogy with the bending of light in 4D Schwarzschild spacetime, to study the bending of "gravitational rays" around the bulk black hole. Since an expanding universe is not described by a Minkowski metric, therefore, Lorenz invariance is already broken by the expansion of the universe. We should rather call such effects "causality violations". However, as explained in ref. [9], the 4D effective action for a field that can propagate through the bulk is not Lorentz invariant, so we call such effects Lorentz invariance violations.

We also address the question whether such apparently superluminal signals can solve the horizon problem in the absence of inflation.

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We find that for both the static and the expanding universes the gravitational wave traveling through the bulk reaches a brane observer before a light wave emitted from the same source as the gravitational wave, but traveling along the brane. Also if our brane separates two different AdSS spaces, there will be two different gravitational waves traveling through the two bulks, each reaching a brane observer at different times, both before a light wave traveling along the brane. Regarding the horizon problem, we find that it is possible for the bulk signals to travel far enough to homogenize the universe we see today. However, the signals are redshifted by the gravitational field of the bulk black hole, and it is unlikely that they will carry enough energy back to the brane to have a significant effect on homogenizing the universe.

2. Bulk null geodesics

2.1 Validity of the light-ray approximation

The light-ray approximation used to calculate the advance of the bulk gravitational waves with respect to the electromagnetic signal traveling along the brane is valid only if the wavelength of the graviton is much smaller than the curvature radius of the AdSS bulk:[1] $\lambda \ll 1/l$. If the wavelength is comparable or larger than the the AdSS curvature radius, then we should study fluctuations around the AdSS background, describing the propagation of a gravitational wave. The effective 4D cosmological constant is given by:

$$\lambda_{\text{eff}} = \frac{\lambda_{\text{brane}}^2}{4} - \frac{1}{l^2}\,. \tag{2.1}$$

If the brane tension is of the order of 1 TeV then the bulk cosmological constant must be of the same order of magnitude in order to give the small observed 4D cosmological constant. Consequently the calculation of the advance of the gravitational signal is valid only for gravitons with energies of the order of 1 TeV, impossible to observe with LIGO.

2.2 Geodesic equations

We study the propagation of a signal through the AdSS bulk, with the aim to find which geodesics will leave the brane and re-intersect it. For simplicity we first study the stationary brane: the position of the brane along the 5th dimension simultaneously satisfies $H(r_b) = 0$ and $\dot{H}(r_b) = 0$ (see appendix A). We use the setup of ref. [9] in which the brane separates two (not necessarily identical) AdSS spaces described by the metric eq. (1.1). The function $f(r)$ has the general form of ref. [19]:[2]

$$f(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2}\,. \tag{2.2}$$

We will see that the existence of Killing vectors makes (at least for the moment) the explicit form of $f(r)$ irrelevant.

[1] I thank É.E. Flanagan for pointing out the limits of the approximation.

[2] As already mentioned, other authors [12, 22] use the more general AdS-RN space time for which the function $f(r)$ has one extra term: $f(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4}$.

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We are interested in finding the null geodesics of the AdSS space-time. We will allow only 3 coordinates to change, namely, t, r and σ. Depending on the curvature of the 3D spatial sections, this metric is of one of the following three forms:

$$\begin{aligned} d\Sigma_k^2 &= d\sigma^2 + h(\sigma)\left(d\theta^2 + \sin^2\theta d\phi^2\right) , \\ h(\sigma) &= \begin{cases} \sin^2(\sigma)\,, & k = +1\,, \\ \sigma^2\,, & k = 0\,, \\ \sinh^2(\sigma)\,, & k = -1\,. \end{cases} \end{aligned} \tag{2.3}$$

Since we keep θ and ϕ constant, the explicit form of $h(\sigma)$ is irrelevant. With this restriction the metric for which we want to find the geodesics is:

$$d\hat{s}^2 = -f(r)dt^2 + r^2 d\sigma^2 + \frac{dr^2}{f(r)}\,. \tag{2.4}$$

We notice that, up to specifying the explicit form of $f(r)$, this metric is identical with the 4D Schwarzschild metric with the restriction $d\phi = 0$.

We could now proceed by writing the geodesic equations,

$$\frac{\partial^2 x^\mu}{\partial\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{\partial x^\alpha}{\partial\lambda}\frac{\partial x^\beta}{\partial\lambda} = 0 \tag{2.5}$$

but we can use the fact that fictitious space-time described by the metric (2.4) possesses the Killing vector fields: $(\partial/\partial t)^\mu$, $(\partial/\partial\sigma)^\mu$. We obtain two prime integrals:

$$\left(\frac{\partial}{\partial t}\right)^\mu \left(\frac{\partial x^\nu}{\partial\lambda}\right) g_{\mu\nu} = \text{constant} \longrightarrow -f(r)\frac{\partial t}{\partial\lambda} = -E\,, \tag{2.6}$$

$$\left(\frac{\partial}{\partial \sigma}\right)^\mu \left(\frac{\partial x^\nu}{\partial\lambda}\right) g_{\mu\nu} = \text{constant} \longrightarrow r^2\frac{\partial r}{\partial\lambda} = P\,. \tag{2.7}$$

One may argue that the original metric does not possess the Killing vector field $(\partial/\partial\sigma)^\mu$, but the same results can be obtained by integrating the geodesic equations. The geodesic equations for the t and σ coordinates are:

$$\frac{d^2 t}{d\lambda^2} + \frac{1}{f(r)}\frac{\partial f(r)}{\partial r}\frac{\partial t}{\partial\lambda}\frac{\partial r}{\partial\lambda} = 0\,, \tag{2.8}$$

$$\frac{d^2\sigma}{d\lambda^2} + \frac{2}{r}\frac{\partial r}{\partial\lambda}\frac{\partial\sigma}{\partial\lambda} = 0\,. \tag{2.9}$$

They can be written in the form:

$$\frac{1}{r^2}\frac{\partial}{\partial\lambda}\left[r^2\frac{\partial\sigma}{\partial\lambda}\right] = 0 \Longrightarrow r^2\frac{\partial\sigma}{\partial\lambda} = \text{constant}\,, \tag{2.10}$$

$$\frac{1}{f(r)}\frac{\partial}{\partial\lambda}\left[f(r)\frac{\partial t}{\partial\lambda}\right] = 0 \Longrightarrow f(r)\frac{\partial t}{\partial\lambda} = \text{constant} \tag{2.11}$$

which coincides with the previous result. Using these two prime integrals and the fact that the tangent vector to the geodesic is a null vector:

$$-f(r)\left(\frac{\partial t}{\partial\lambda}\right)^2 + r^2 d\left(\frac{\partial\sigma}{\partial\lambda}\right)^2 + \frac{1}{f(r)}\left(\frac{\partial r}{\partial\lambda}\right)^2 = 0 \tag{2.12}$$

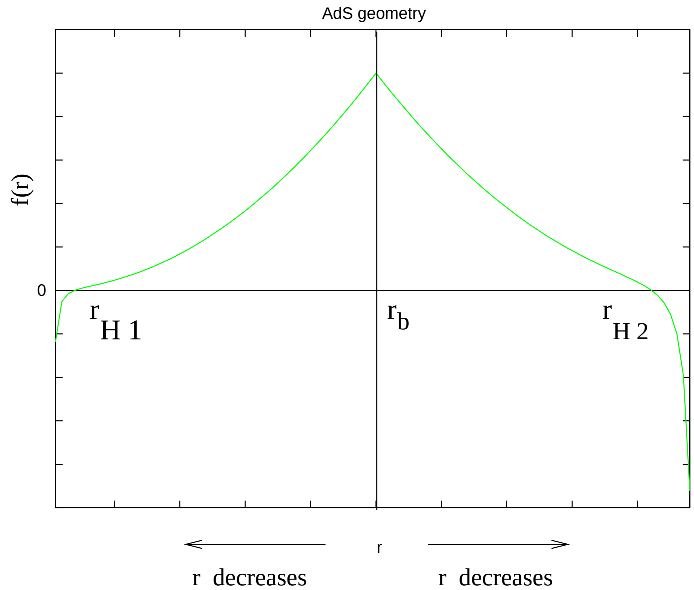

Figure 1: The picture shows the warp factor as a function of the radial coordinate. The brane is a domain wall placed at r_b, separating two (generically different) AdSS spaces with horizons at r_{H1} and r_{H2}, so signals can travel outside the brane through either space.

we obtain the following prime integrals that allow us to discuss the properties of the geodesics.

$$\frac{\partial t}{\partial \lambda} = \frac{E}{f(r)}, \tag{2.13}$$

$$\frac{\partial \sigma}{\partial \lambda} = \frac{P}{r^2}, \tag{2.14}$$

$$\frac{\partial r}{\partial \lambda} = \pm E\sqrt{1 - \frac{P^2}{E^2}\frac{f(r)}{r^2}}\,. \tag{2.15}$$

Using the analogy with the bending of light calculations, in figure 2 we represent σ as an angular variable. Since in the present setup (see ref. [9]) the coordinate r decreases on either side of the brane, we are only interested in geodesics that are contained in the space delimited by the brane and the horizon, as illustrated in figure 1.

We can now proceed to analyze the propagation of the bulk null signals and find out how they are perceived by a brane observer. More precisely we want to find which geodesics can start on the brane, travel through the bulk, and re-intersect the brane. Then we find the separation of the two events (emission, re-entry) as seen by a brane observer.

3. Brane re-intersecting geodesics

We now want to find those geodesics that will be emitted from the brane, travel through the bulk and re-intersect the brane. Since the r coordinate decreases on both sides of the brane, the geodesics will first move toward decreasing r so in eq. (2.15) we have to

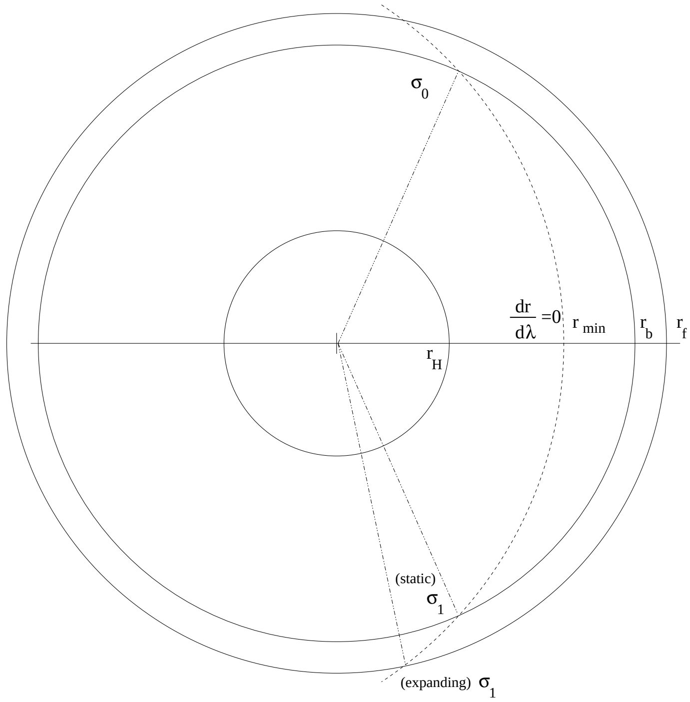

Figure 2: The brane is located at $r = r_b$ for the static case, or moves from r_b to r_f in the expanding case. The geodesics that re-intersect the brane must satisfy the condition $r_H < r_{\min}$, r_H being the radius of the AdSS horizon, and $r_{\min}$ the location of the turning point.

choose the "$-$" sign. The brane is either located at a fixed r, or moving toward larger r (corresponding to an expanding universe), so in order for the geodesics to re-intersect the brane they have to reach a turning point after which they move toward increasing r. The movement toward increasing r corresponds to the "+" sign in eq. (2.15) so in order to switch between the initial "$-$" sign to the "+" sign, $\partial r/\partial\lambda$ must be zero at the turning point. Moreover, the turning point must be outside the horizon of the bulk black hole, since it will take null signals an infinite coordinate time to reach the horizon, so these signals do not re-intersect the brane.

We will study the zeroes of the function: $F(r) = 1 - \frac{P^2}{E^2}\frac{f(r)}{r^2}$ since these zeroes give the location of the turning point of the bulk null geodesics. The equation becomes:

$$1 - \frac{P^2}{E^2}\left(\frac{1}{l^2} + \frac{k}{r^2} - \frac{\mu}{r^4}\right) = 0\,. \tag{3.1}$$

First, the locations of the AdSS horizons are given by the zeroes of the function $f(r) = k + r^2/l^2 - \mu/r^2$. The solutions are:

$$\frac{1}{r_H^2} = \frac{k \mp \sqrt{k^2 + 4\frac{\mu}{l^2}}}{2\mu}\,. \tag{3.2}$$

If such a horizon exists, the brane is located in the region where $f(r) > 0$. We can see the location of the zeroes of $f(r)$ in figure 3. Unless $\mu < 0$ there is only one horizon

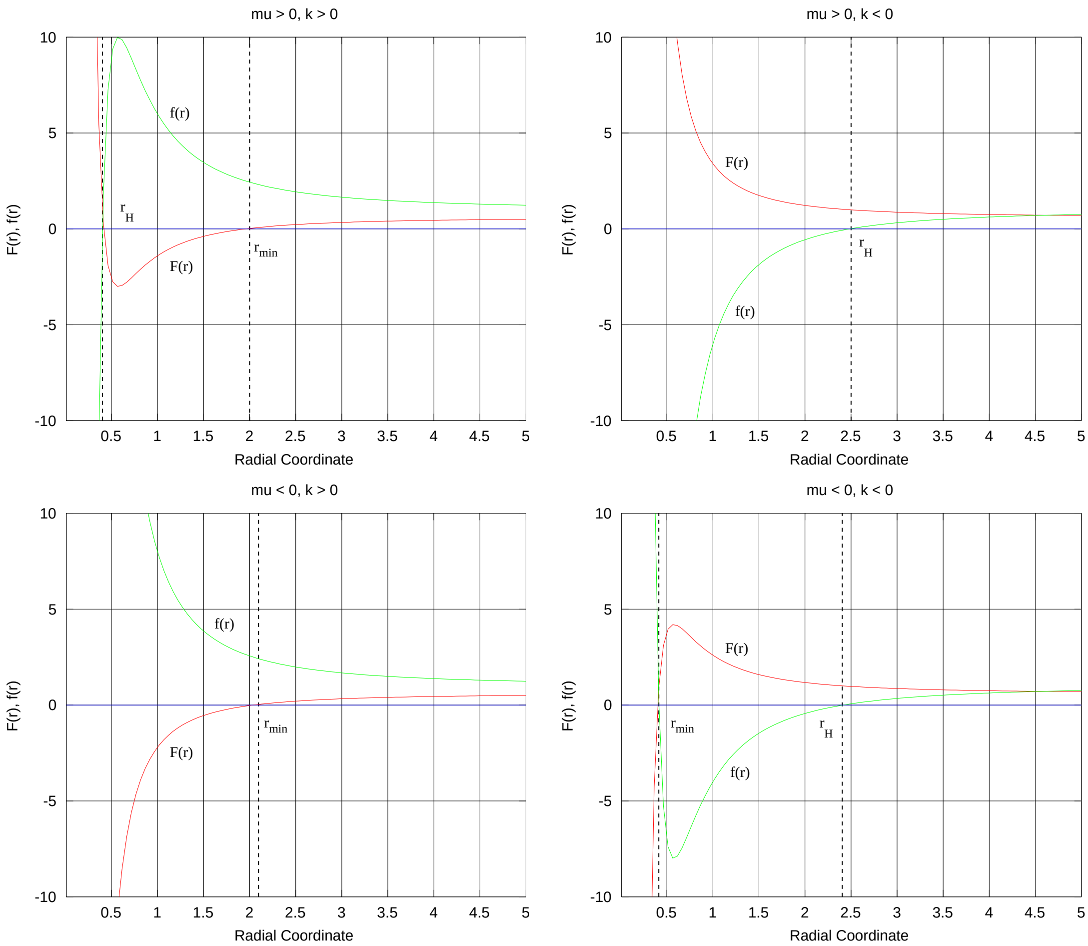

Figure 3: The positions of the horizons are given by the zeros of $f(r)$, and the positions of the turning points by the zeros of $F(r)$. Their locations are marked by vertical dashed lines. The functions $f(r)$ and $F(r)$ are plotted for all possible signatures of μ and k.

corresponding to the "+" solution (the "−" solution corresponds to $r_H^2 < 0$). The zeroes of $F(r)$ are given by:

$$\frac{1}{r_{\rm min}^2} = \frac{k \mp \sqrt{k^2 + 4\mu\left(\frac{1}{l^2} - \frac{E^2}{P^2}\right)}}{2\mu}. \tag{3.3}$$

We will now discuss each signature of μ and k. All cases are illustrated in figure 3.

- $\mu > 0$, $k > 0$ in this case there is only one horizon corresponding to the "+" solution. Depending on the value of P^2/E^2, $F(r)$ can have two zeros outside the horizon, in which case the geodesic corresponding to the particular value of P^2/E^2 will return at the outer one (the "+" solution). If the two zeroes merge we will see that the corresponding geodesic will not return to the brane. There will also be geodesics that do not return to the brane, because for small enough P^2/E^2, $F(r)$ will have no zeros. $F(r)$ will have no zeroes if it is positive at its minimum. This gives the condition: $P^2/E^2 \leq 1/(1/l^2 + k^2/4\mu)$

- $\mu > 0$, $k < 0$ there is one horizon corresponding to the "+" solution. The geodesics either have no turning point, or are not emitted at all. We can find values of P^2/E^2 such that $F(r)$ has a zero, but if $r_{\min}$ satisfies the condition, $r_H < r_{\min} < r_b$, then $F(r)$ will be negative at r_b, meaning that the geodesics corresponding to the values of P^2/E^2 do not intersect the brane at all.

- $\mu < 0$, $k > 0$ there is no horizon in this case, only a naked singularity at $r = 0$. In this case all geodesics will have a turning point.

- $\mu < 0$, $k < 0$ depending on the values of μ, k and l, there can be two horizons, one horizon, or no horizon. Let us look at the solution eq. (3.2) and rewrite it in terms of the absolute values of μ and k.

$$\frac{1}{r_H^2} = \frac{|k| \pm \sqrt{k^2 - 4\frac{|\mu|}{l^2}}}{2\mu} . \tag{3.4}$$

Solutions exist only if $|\mu| \leq k^2 l^2/4$. Strict inequality corresponds to the existence of two horizons, only the outer one (the "+" solution) being of interest to us.

  - If $|\mu| > k^2 l^2/4$, there is a naked singularity at $r = 0$ and all the geodesics will have a turning point.
  - If $|\mu| = k^2 l^2/4$ the two horizons merge and are located at $r_H = \sqrt{2|\mu|/|k|}$. If $1 - (P^2/E^2)1/l^2 \geq 0$, there will be one turning point located behind the horizon, so no geodesic returns. If $1 - (P^2/E^2)1/l^2 < 0$ there will be two turning points, one outside and one inside the horizon. However, if we have $r_H < r_{\min}$, then $F(r_b) < 0$, so the corresponding geodesics do not intersect the brane at all.
  - If $|\mu| < k^2 l^2/4$ there will be two horizons. $F(r)$ will have one or two zeros, depending on the value of P^2/E^2. For $r \to 0$, $F(r)$ is dominated by $-(P^2/E^2)(|\mu|/r^4)$ which is negative, and for $r \to \infty$ by $1 - (P^2/E^2)1/l^2$ which can be positive or negative. The function has an extremum at $r^2 = 2|\mu|/|k|$. The second derivative evaluated at the extremum is equal to $-(P^2/E^2)(|k|^3/|\mu|^2)$, so the extremum is a *maximum*. We thus find: If $1 - (P^2/E^2)1/l^2 > 0$ the geodesics will not return to the brane since the turning point is located behind the outer horizon. If $1 - (P^2/E^2)1/l^2 < 0$, depending on the value of P^2/E^2, there will be two turning points, one turning point, or no turning points depending on whether $1 - (P^2/E^2)(1/l^2 - k^2/4|\mu|)$ is positive, zero, or negative. If there is only one turning point, then the maximum of $F(r)$ is equal to zero, so $F(r_b) < 0$: the corresponding geodesic does not intersect the brane. If there are two turning points, one can be outside the horizon, but if $r_{\min}$ of the outer turning point satisfies $r_H < r_{\min} < r_b$, then $F(r_b) < 0$ so the corresponding geodesic does not intersect the brane at all.

4. Static brane

We will now assume that the function $F(r)$ has a zero close to r_b, such that:

$$\frac{|r_b - r_{\min}|}{r_b} \ll 1\,. \tag{4.1}$$

In this case we will expand the function $F(r)$ around this zero:

$$F(r) = [F(r_{\min}) = 0] + \left.\frac{dF}{dr}\right|_{r=r_{\min}} \delta r + \frac{1}{2}\left.\frac{d^2F}{dr^2}\right|_{r=r_{\min}} (\delta r)^2 + O\left(\delta r^3\right). \tag{4.2}$$

Using the eqs. (2.13), (2.14) and (2.15) we can eliminate the affine parameter λ and obtain:

$$dt = \frac{dr}{f(r)\sqrt{F(r)}}\,, \qquad d\sigma = \frac{P}{E}\frac{dr}{\sqrt{F(r)}}\,. \tag{4.3}$$

Writing $r = r_{\min} + \delta r$ the coordinate time and distance it takes a signal to leave and re-entry the brane is given by the sum of two integrals:

$$\Delta t, \sigma = \int_{r_b}^{r_{\min}} dt, \sigma + \int_{r_{\min}}^{r_f} dt, \sigma\,. \tag{4.4}$$

Since we chose the brane to be static, $r_f = r_b$:

$$\Delta t = 2\int_0^{r_b - r_{\min}} \frac{d(\delta r)}{f(r_{\min} + \delta r)\sqrt{F(r_{\min} + \delta r)}}\,, \tag{4.5}$$

$$\Delta \sigma = 2\frac{P}{E}\int_0^{r_b - r_{\min}} \frac{d(\delta r)}{(r_{\min} + \delta r)^2\sqrt{F(r_{\min} + \delta r)}}\,. \tag{4.6}$$

Substituting in the above equations the series expansion eq. (4.2) we observe that the integrals are convergent if $\frac{dF}{dr}|_{r=r_{\min}} \neq 0$, but are divergent if $\frac{dF}{dr}|_{r=r_{\min}} = 0$. In this case it will take an infinite time for the signals to return (see appendix B). For the case $\frac{dF}{dr}|_{r=r_{\min}} \neq 0$ we see from eq. (2.15) that $F(r)$ is positive between $r_{\min}$ and r_b and since $F(r_{\min}) = 0$ we must have $\frac{dF}{dr}|_{r=r_{\min}} > 0$. The function $f(r)$ has zeroes only at the location of the AdSS horizons, so $f(r_{\min}) > 0$.We can now use the expansion eq. (4.2) and keep only the leading order term when calculating Δt and $\Delta\sigma$.

$$\Delta t = 2\int_0^{r_b - r_{\min}} \frac{d(\delta r)}{f(r_{\min})\sqrt{F'(r_{\min})}\sqrt{\delta r}} = \frac{4\sqrt{r_b - r_{\min}}}{f(r_{\min})\sqrt{F'(r_{\min})}}\,, \tag{4.7}$$

$$\Delta \sigma = 2\frac{P}{E}\int_0^{r_b - r_{\min}} \frac{d(\delta r)}{(r_{\min})^2\sqrt{F'(r_{\min})}\sqrt{\delta r}} = \frac{4P}{E}\frac{4\sqrt{r_b - r_{\min}}}{(r_{\min})^2\sqrt{F'(r_{\min})}}. \tag{4.8}$$

The induced metric on the brane will be:

$$ds_b^2 = -f(r_b)dt^2 + r_b^2 d\sigma^2\,. \tag{4.9}$$

Notice that we kept the bulk coordinates to express distances on the brane, even though they are not the natural coordinates for the brane observer, since it will be easier to find the separation of the emission and the re-entry. The coefficients of the metric are constant, consequently the separation of the emission and re-entry will be:

$$\Delta s^2 = -f(r_b)\Delta t^2 + r_b^2 \Delta\sigma^2 \,. \tag{4.10}$$

Using the above results for Δt and $\Delta\sigma$ we want to find whether $\Delta s^2 > 0$ or $\Delta s^2 < 0$. For calculational convenience we will work with the ratio $\Delta s^2 / r_b^2 \Delta\sigma^2$.

$$\begin{aligned}\frac{\Delta s^2}{r_b^2 \Delta\sigma^2} &= 1 - \frac{f(r_b)}{r_b^2}\left(\frac{\Delta t}{\Delta\sigma}\right)^2 \\ &= 1 - \frac{f(r_b)}{r_b^2}\frac{\left[\frac{1}{f(r_{\min})\sqrt{F'(r_{\min})}}\right]^2}{\left[\frac{P}{E}\frac{1}{(r_{\min})^2\sqrt{F'(r_{\min})}}\right]^2} = 1 - \frac{f(r_b)}{r_b^2}\frac{E^2}{P^2}\frac{r_{\min}^4}{f^2(r_{\min})} \,. \end{aligned} \tag{4.11}$$

We use the fact that

$$F(r_{\min}) = 1 - \frac{P^2}{E^2}\frac{f(r_{\min})}{r_{\min}^2} = 0 \implies \frac{r_{\min}^2}{f(r_{\min})} = \frac{P^2}{E^2} \tag{4.12}$$

to obtain:

$$\frac{\Delta s^2}{r_b^2 \Delta\sigma^2} = 1 - \frac{f(r_b)}{r_b^2}\frac{E^2}{P^2}\frac{P^4}{E^4} = 1 - \frac{f(r_b)}{r_b^2}\frac{P^2}{E^2} = F(r_b) > 0 \,. \tag{4.13}$$

We obtained that $\Delta s^2 > 0$, consequently the brane observer will see the two events as spatially separated, so the bulk signal travels faster than light on the brane.

We now try to express the ratio $\Delta s^2 / r_b^2 \Delta\sigma^2$ in terms of the scale factor of the visible universe, r_b, and the physical distance separating the two events as seen by a brane observer. Using the solution (4.8), and expressing P^2/E^2 in terms of $r_{\min}$, we obtain:

$$\Delta\sigma = \frac{4\sqrt{r_b - r_{\min}}}{r_{\min}\sqrt{\frac{2f(r_{\min})}{r_{\min}} - f'(r_{\min})}} \,. \tag{4.14}$$

We see that we can now express $r_{\min}$ in terms of $\Delta\sigma$ and r_b, so we can express the relative advance of the gravitational signal in terms of the physical separation of the emission and re-entry points on the brane, $r_b \Delta\sigma$, and the parameters of the AdSS bulk, namely, k, l and μ. Using eq. (4.13) and the fact that $F(r_{\min}) = 0$, we obtain:

$$\frac{\Delta s^2}{r_b^2 \Delta\sigma^2} = F'(r_{\min})(r_b - r_{\min}) + O(r_b - r_{\min})^2 \,. \tag{4.15}$$

Now we can further use the fact that $r_b - r_{\min}$ is a small quantity and write:

$$F'(r_{\min}) = F'(r_b) + F''(r_b)(r_b - r_{\min}) + O(r_b - r_{\min})^2 \tag{4.16}$$

so we obtain (up to corrections of order $O(r_b - r_{\min})^2$)

$$\frac{\Delta s^2}{r_b^2 \Delta\sigma^2} \approx F'(r_b)(r_b - r_{\min}) \approx \left(\frac{2}{r_b} - \frac{f'(r_b)}{f(r_b)}\right)(r_b - r_{\min}) \,. \tag{4.17}$$

We now use eq. (4.14) and again the fact that $r_b - r_{\min}$ is a small quantity:

$$\Delta\sigma = \frac{4\sqrt{r_b - r_{\min}}}{r_{\min}\sqrt{f(r_{\min})}\sqrt{\frac{2}{r_{\min}} - \frac{f'(r_{\min})}{f(r_{\min})}}} \approx \frac{4\sqrt{r_b - r_{\min}}}{r_b\sqrt{f(r_b)}\sqrt{\frac{2}{r_b} - \frac{f'(r_b)}{f(r_b)}}}\,. \tag{4.18}$$

We obtain:

$$r_b - r_{\min} = \left(\frac{\Delta\sigma}{4}\right)^2 r_b^2 f(r_b)\left(\frac{2}{r_b} - \frac{f'(r_b)}{f(r_b)}\right) \tag{4.19}$$

and the final form for the relative advance of the gravitational signal with respect to the light signal along the brane:

$$\frac{\Delta s^2}{r_b^2\Delta\sigma^2} \approx \frac{1}{16} f(r_b)\left(\frac{2}{r_b} - \frac{f'(r_b)}{f(r_b)}\right)^2 (\Delta L)^2\,, \tag{4.20}$$

where $\Delta L = r_b\Delta\sigma$ is the physical separation between the emission and re-entry points of the gravitational signal.

5. Expanding brane

We will now consider the case where the 4D Hubble constant is non zero. We obtain a non-zero 4D Hubble constant by allowing the brane to move along the 5th dimension, $r_b = r_b(t)$. We want to find the proper time of an observer moving with the brane. We therefore choose an observer whose world-line is described by $\sigma, \theta, \phi =$ constant. Following ref. [13] we express the proper time of the brane in terms of the $5D$ coordinate time.

$$ds^2_{\text{brane}} = -d\tau^2 = -f(r_b(t))dt^2 + \frac{dr_b^2}{f(r_b(t))} = -f(r_b(t))dt^2 + \frac{\dot{r}_b^2 d\tau^2}{f(r_b(t))}\,, \tag{5.1}$$

where the dot represents the derivative taken with respect to the proper time of the brane τ. The induced metric on the brane becomes:

$$ds^2_{\text{brane}} = -d\tau^2 + r_b^2(\tau)d\Sigma_k^2 \tag{5.2}$$

so the Hubble constant seen by a brane observer is: $H = \dot{r}_b/r_b$. We can now use eq. (5.1) to express dr_b/dt as a function of $H = \dot{r}_b/r_b$.

$$f(r_b(t))dt^2 = d\tau^2\left(1 + \frac{\dot{r}_b^2}{f(r_b(t))}\right) = d\tau^2\left(1 + \frac{H^2 r_b^2}{f(r_b(t))}\right)\,, \tag{5.3}$$

$$\frac{dr_b}{dt} = \frac{dr_b}{d\tau}\frac{d\tau}{dt} = \dot{r}_b\sqrt{\frac{f(r_b)}{1 + \frac{H^2 r_b^2}{f(r_b)}}} = Hr_b\sqrt{\frac{f(r_b)}{1 + \frac{H^2 r_b^2}{f(r_b)}}} = \frac{Hr_b f(r_b)}{\sqrt{f(r_b) + H^2 r_b^2}}\,. \tag{5.4}$$

We can now compare the movement of the brane with the movement of bulk null signals so that we can identify the signals that will re-entry the brane. For these signals, since the 4D metric is no longer time-independent, we will calculate the coordinate distance traveled by a null signal along the brane, and compare it with the coordinate distance along the brane traveled by the bulk signal.

$$\Delta\sigma_{\text{brane null}} = \int_{r_{\text{initial}}}^{r_{\text{final}}} \frac{d\tau}{r_b(\tau)} = \int_{r_{\text{initial}}}^{r_{\text{final}}} \frac{dr_b}{Hr_b^2}\,. \tag{5.5}$$

We will assume that the Hubble constant is (approximately) a true constant, namely that r_b is sufficiently large that between the emission and re-entry of the bulk signal the amount of brane expansion satisfies the condition:

$$\frac{\Delta r_b}{r_b} \ll 1\,. \tag{5.6}$$

Also for large r_b the function $f(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2}$ is dominated by the term proportional to r^2, so that for the motion of the brane we obtain the equation:

$$\frac{dr_b}{dt} = \frac{Hr_b f(r_b)}{\sqrt{f(r_b) + H^2 r_b^2}} \approx \frac{Hr_b^3/l^2}{\sqrt{\left(H^2 + \frac{1}{l^2}\right) r_b^2}} = \frac{Hr_b^2}{l\sqrt{1+H^2l^2}}\,. \tag{5.7}$$

The resulting equation for the evolution of the position of the brane is easy to integrate:

$$\frac{dr_b}{r_b^2} = \frac{H}{l}\frac{dt}{\sqrt{1+H^2l^2}} \implies \frac{1}{r_0} - \frac{1}{r_f} = \frac{H}{l}\frac{t}{\sqrt{1+H^2l^2}}\,. \tag{5.8}$$

The equation of motion for null signals through the bulk is given by:

$$\frac{dr}{dt} = \pm f(r)\sqrt{F(r)}\,. \tag{5.9}$$

Initially r will decrease to $r_{\min}$ ($F(r_{\min}) = 0$), so we will have to use the "$-$" solution. After reaching $r_{\min}$, r will increase, so we will have to consider the "+" solution. We will continue to use the the expansion around $r_{\min}$, so we can use the previous result eq. (4.7):

$$\Delta t = \frac{2\sqrt{r_0 - r_{\min}}}{f(r_{\min})\sqrt{F'(r_{\min})}} + \frac{2\sqrt{r_f - r_{\min}}}{f(r_{\min})\sqrt{F'(r_{\min})}}\,, \tag{5.10}$$

where r_f is the r-coordinate where the bulk signal re-enters the brane. The time coordinate for the re-entry of the bulk signal is the intersection of the trajectories described by the eqs. (5.7) and (5.9), having the initial conditions $r_b(0) = r_0$ and $r(0) = r_0$. So we will set r_f to be the same in both eqs. (5.8) and (5.10) and solve for t. We use the fact that the relative increase in r_b is small to linearize eq. (5.7).

$$\frac{1}{r_0} - \frac{1}{r_f} \approx \frac{r_f - r_0}{r_0^2} = \frac{H}{l}\frac{\Delta t}{\sqrt{1+H^2l^2}}\,. \tag{5.11}$$

At the same time we rewrite eq. (5.10) in the form:

$$\sqrt{r_f - r_{\min}} = \frac{f(r_{\min})\sqrt{F'(r_{\min})}\Delta t}{2} - \sqrt{r_0 - r_{\min}} \tag{5.12}$$

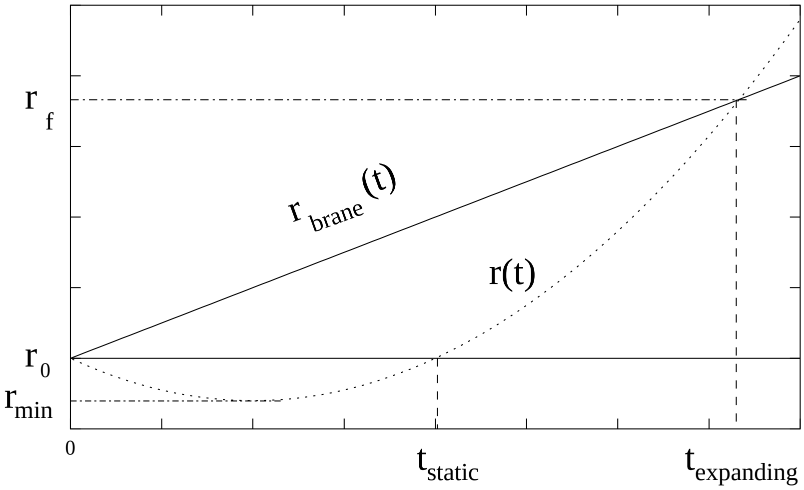

Figure 4: The intersection of the two trajectories $r(t)$ and $r_{\text{brane}}(t)$ is the re-entry time for the bulk signal. We observe that for $H = 0$ we reproduce the result from the static case. t_{static} and $t_{\text{expanding}}$ are the re-entry times for the static and the expanding cases.

so we obtain:

$$\begin{aligned} r_f - r_0 &= \left(\frac{f(r_{\min})\sqrt{F'(r_{\min})}\Delta t}{2} - \sqrt{r_0 - r_{\min}} \right)^2 - (r_0 - r_{\min}) \\ &= \frac{f(r_{\min})\sqrt{F'(r_{\min})}\Delta t}{2} \left(\frac{f(r_{\min})\sqrt{F'(r_{\min})}\Delta t}{2} - 2\sqrt{r_0 - r_{\min}} \right) \\ &= r_0^2 \frac{H}{l} \frac{\Delta t}{\sqrt{1 + H^2 l^2}} \end{aligned} \tag{5.13}$$

so we obtain the re-entry time for the bulk signal:

$$\Delta t = \frac{4\sqrt{r_0 - r_{\min}}}{f(r_{\min})\sqrt{F'(r_{\min})}} + \frac{4r_0^2 H}{f^2(r_{\min})F'(r_{\min})l\sqrt{1 + H^2 l^2}} \,. \tag{5.14}$$

The solution can be seen in figure 4.

We now go back and calculate $\Delta\sigma$ for the brane signal and for the bulk signal. Again, we assume that $H \approx$ constant and that r is large so that the relative expansion of the brane is small, $\Delta r_b / r_b \ll 1$, and $f(r) \approx r^2/l^2$. For the brane null signal we have:

$$\Delta\sigma_{\text{brane null}} = \int_{r_0}^{r_f} \frac{dr_b}{H r_b^2} \approx \frac{1}{H} \left(\frac{1}{r_0} - \frac{1}{r_f} \right) . \tag{5.15}$$

We use now eq. (5.8) to obtain:

$$\Delta\sigma_{\text{brane null}} = \int_{r_0}^{r_f} \frac{dr_b}{H r_b^2} \approx \frac{1}{H} \left(\frac{1}{r_0} - \frac{1}{r_f} \right) = \frac{1}{H} \frac{H}{l} \frac{\Delta t}{\sqrt{1 + H^2 l^2}} = \frac{1}{\sqrt{1 + H^2 l^2}} \frac{1}{l} \Delta t \,. \tag{5.16}$$

For the bulk null signal we have:

$$\Delta\sigma_{\text{bulk null}} = -\frac{P}{E}\int_{r_0}^{r_{\min}} \frac{d(\delta r)}{(r_{\min})^2\sqrt{F'(r_{\min})}\sqrt{\delta r}} + \frac{P}{E}\int_{r_{\min}}^{r_f} \frac{d(\delta r)}{(r_{\min})^2\sqrt{F'(r_{\min})}\sqrt{\delta r}}$$
$$\frac{2P}{E}\frac{1}{(r_{\min})^2\sqrt{F'(r_{\min})}}(\sqrt{r_0 - r_{\min}} + \sqrt{r_f - r_{\min}}) = \frac{P}{E}\frac{f(r_{\min})}{(r_{\min})^2}\Delta t\,. \tag{5.17}$$

Using the fact that $P^2/E^2 = (r_{\min})^2/f(r_{\min})$, and the approximation $f(r) \approx r^2/l^2$ we obtain:

$$\Delta\sigma_{\text{bulk null}} = \frac{1}{l}\Delta t\,. \tag{5.18}$$

Comparing this result with the one from eq. (5.16) we obtain immediately that:

$$\Delta\sigma_{\text{brane null}} < \Delta\sigma_{\text{bulk null}} \tag{5.19}$$

so the bulk signal will appear as superluminal to the brane observer. We see now why we should consider the static and expanding brane cases separately; taking the limit $H \to 0$ in eq. (5.16) would give $\Delta\sigma_{\text{brane null}} = \Delta\sigma_{\text{bulk null}}$, but the limit is meaningless since $\Delta\sigma_{\text{brane null}}$ given by eq. (5.16) naively diverges. We should take into account that as $H \to 0$ then $r_f \to r_0$ and the combination $1/H(1/r_0 - 1/r_f)$ is finite.

6. The horizon problem

We want to address the question whether the apparently superluminal signals can solve the horizon problem in the absence of inflation. Inflationary models assume that the universe starts in a radiation-dominated phase during which it can become homogeneous, followed by the inflationary phase, when the comoving horizon, defined as $1/HR(\tau)$, shrinks by a factor of at least e^{55}. After the end of inflation, the comoving horizon increases again, but regions that were causally disconnected during inflation look identical (on sufficiently large scales) because they were in causal contact before the inflationary epoch.

In order for the bulk signals to homogenize the universe on comoving scales that become visible today, the signals should bridge distances much larger than the size of the horizon at the moment of emission. Moreover, we expect such signals to be emitted when the temperature was larger than $\sim 1\,\text{TeV}$ and re-enter the visible universe before the recombination epoch. The reason is that once structure (galaxy clusters) forms, such signals should have little effect on homogenizing the universe, as the re-entry of a bulk signal should manifest itself as a flash of energy coming from the decay of an invisible particle. It is hard to imagine how such processes can generate a homogeneous universe without wiping out any structure already formed. Therefore we consider only signals emitted when $T \sim 1\,\text{TeV}$ and re-enter the universe the latest when $T \sim 0.2\,\text{eV}$, corresponding to the recombination epoch.

However, when the temperature is $T \sim 1\,\text{TeV}$ the universe is radiation-dominated and expands rapidly, and in our model the expansion is described by the movement of the brane away from the bulk black hole. The signals are emitted from the brane close to the bulk black hole and will re-enter the brane farther away from it, so they are red-shifted in the gravitational field of the bulk black hole. Moreover the signals are emitted from a moving brane, so we should take into account the additional Doppler shift.

We now address the question whether it is possible for bulk signals to travel far enough to homogenize the visible universe. In order to do so we analyze the travel of a signal moving at the speed of light along the brane. The signal was emitted when the temperature was $\sim$ 1 TeV, and we want to find the comoving distance it travels until the present epoch. If there is no inflation, there are two relevant parts of the journey: In the first phase the universe is radiation-dominated, and after matter-radiation decoupling the universe becomes matter-dominated. For the radiation-dominated universe the energy density decreases with the expansion like $\rho \sim 1/R^4$ while for the matter-dominated era $\rho \sim 1/R^3$.

For a null signal traveling along the brane have:

$$ds^2 = -d\tau^2 + R^2(\tau)d\sigma^2 = 0 \Longrightarrow d\sigma = \frac{d\tau}{R(\tau)} = \frac{dR}{HR^2(\tau)} \,. \tag{6.1}$$

The Hubble constant is given by:

$$H^2 = \frac{8\pi G}{3}\rho = \begin{cases} \sim 1/R^4, & \text{radiation dominated universe}\,, \\ \sim 1/R^3, & \text{matter dominated universe} \end{cases} \tag{6.2}$$

so that we obtain the comoving distance traveled in terms of the scale factor of the universe (or redshift):

$$\sigma - \sigma_0 = \begin{cases} \frac{R-R_0}{\sqrt{8\pi G\rho_0 R_0^4}}\,, & \text{radiation dominated universe}\,, \\ \frac{\sqrt{R}-\sqrt{R_0}}{\sqrt{8\pi G\rho_0 R_0^3}}\,, & \text{matter dominated universe}\;. \end{cases} \tag{6.3}$$

The decoupling of radiation from matter occurs at a redshift of 10^3, while the emission of the signal occurs at a redshift of 10^{16}, corresponding to a temperature of 1 TeV. Therefore most of the comoving distance traveled by the null signal along the brane comes from the matter-dominated era. At the same time, the bulk signal must travel at least the same comoving distance and re-enter the brane before decoupling, namely at a redshift $> 10^3$. The equation describing the movement of the bulk signal is:

$$d\sigma = \frac{P}{E}\frac{dr}{\sqrt{F(r)}} \tag{6.4}$$

and for large values of R the comoving distance traveled by the bulk signal is:

$$\sigma - \sigma_0 = \frac{R - R_0}{\sqrt{\frac{E^2}{P^2} - \frac{1}{l^2}}} \,. \tag{6.5}$$

We see that for the radiation dominated universe both comoving distances depend linearly on the scale factor, while for the matter-dominated universe the bulk signal can travel "faster" than the signal traveling along the brane. We can always choose E/P such that the denominator of eq. (6.5) is small and the comoving distance traveled by the bulk signal exceeds the one traveled by the brane signal by a large factor. It is therefore possible for the bulk signal to travel the comoving distance required to homogenize the visible universe. However, we must still address the question whether the signal will deposit a sizable amount of energy on the brane upon re-entry.

6.1 Red-shift of the bulk signal

In order to study the emission and re-entry of the signal we use the equation of motion of the brane and the equation of motion of the bulk signal:

$$\frac{dr_b}{dt} = \frac{H r_b f(r_b)}{\sqrt{f(r_b) + H^2 r_b^2}}\,, \qquad \frac{dr}{dt} = \pm f(r)\sqrt{F(r)}\,. \tag{6.6}$$

The signal is emitted when the brane is located at $r_{\rm em}$ and re-enters (it is absorbed) when the brane is located at $r_{\rm ab}$. We integrate the above equations:

$$t_{\rm ab} - t_{\rm em} = \int_{r_{\rm em}}^{r_{\rm ab}} \frac{\sqrt{f\,(r_b) + H^2 r_b^2}}{H r_b f\,(r_b)} dr_b\,, \tag{6.7}$$

$$t_{\rm ab} - t_{\rm em} = \int_{r_{\rm em}}^{r_{\rm min}} -\frac{dr}{f(r)\sqrt{F(r)}} + \int_{r_{\rm min}}^{r_{\rm ab}} \frac{dr}{f(r)\sqrt{F(r)}}\,. \tag{6.8}$$

We now consider two signals emitted along the same trajectory at a short time interval one after the other, more precisely, at moments $t_{\rm em}$ and $t_{\rm em} + \delta t_{\rm em}$. We want to find the time interval between the re-entry of the two signals, so we denote the re-entry moments by $t_{\rm ab}$ and $t_{\rm ab} + \delta t_{\rm ab}$. We can use any one of eqs. (6.7) and (6.8), but we prefer eq. (6.8) because the Doppler effect mentioned before is readily visible.

$$(t_{\rm ab} + \delta t_{\rm ab}) - (t_{\rm em} + \delta t_{\rm em}) = \int_{r_{\rm em}+\delta r_{\rm em}}^{r_{\rm min}} -\frac{dr}{f(r)\sqrt{F(r)}} + \int_{r_{\rm min}}^{r_{\rm ab}+\delta r_{\rm ab}} \frac{dr}{f(r)\sqrt{F(r)}}\,. \tag{6.9}$$

Subtracting eq. (6.8) from eq. (6.9), we obtain:

$$\delta t_{\rm ab} - \delta t_{\rm em} = \frac{\delta r_{\rm em}}{f(r_{\rm em})\sqrt{F(r_{\rm em})}} + \frac{\delta r_{\rm ab}}{f(r_{\rm ab})\sqrt{F(r_{\rm ab})}}\,. \tag{6.10}$$

We can understand the meaning of the above equation by looking at figure 2. The induced metric on the brane is $ds^2 = -d\tau^2 + R^2(\tau)d\sigma^2$ and since the brane expands, in the $5D$ description it moves towards increasing R, i.e. away from the black hole. The signal is first emitted towards the bulk black hole, and in the direction opposite to the movement of the brane. This results in the first red-shift term in eq. (6.9). When the signal re-enters the brane, both the brane and the signal move towards increasing R, which results in the second red-shift term.

We now want to relate $\delta t_{\rm ab}$, $\delta t_{\rm em}$ and $\delta r_{\rm ab}$, $\delta r_{\rm em}$ with the proper time of a brane observer. By doing so we get an estimate of the red-shift of the signal as seen by a brane observer. The coordinate time, t, and the proper time of a brane observer, τ, are related by:

$$dt = d\tau\sqrt{\frac{1}{f(r_b)}\left(1 + \frac{H^2 r_b^2}{f(r_b)}\right)}\,. \tag{6.11}$$

We can use the definition of the Hubble constant to relate $\delta r_{\rm ab}$, $\delta r_{\rm em}$, with the proper time of a brane observer.

$$H = \frac{1}{R}\frac{dR}{d\tau} \Longrightarrow \delta r_{\rm em} = H_{\rm em} r_{\rm em} \delta\tau_{\rm em}\,, \qquad \delta r_{\rm ab} = H_{\rm ab} r_{\rm ab} \delta\tau_{\rm ab}\,. \tag{6.12}$$

We now use these results in eq. (6.10) and obtain:

$$\frac{d\tau_{\rm ab}}{d\tau_{\rm em}} = \frac{\sqrt{\frac{1}{f(r_{\rm em})}\left(1+\frac{H_{\rm em}^2 r_{\rm em}^2}{f(r_{\rm em})}\right)} + \frac{H_{\rm em} r_{\rm em}}{f(r_{\rm em})\sqrt{F(r_{\rm em})}}}{\sqrt{\frac{1}{f(r_{\rm ab})}\left(1+\frac{H_{\rm ab}^2 r_{\rm ab}^2}{f(r_{\rm ab})}\right)} - \frac{H_{\rm ab} r_{\rm ab}}{f(r_{\rm ab})\sqrt{F(r_{\rm ab})}}}\,. \tag{6.13}$$

We see that in the absence of expansion ($H = 0$), we obtain the usual result for the red-shift of a signal in the gravitational field of a black hole:

$$\frac{d\tau_{\rm ab}}{d\tau_{\rm em}} = \sqrt{\frac{f(r_{\rm ab})}{f(r_{\rm em})}}\,. \tag{6.14}$$

Assuming that both $r_{\rm em}$ and $r_{\rm ab}$ are large compared with the radius of the bulk black hole such that $f(r) \sim r^2/l^2$, and that the expansion rate is small compared to the radial speed of light we obtain:

$$\frac{d\tau_{\rm ab}}{d\tau_{\rm em}} \sim \frac{r_{\rm ab}}{r_{\rm em}} \tag{6.15}$$

namely, the bulk signal is red-shifted exactly like radiation on the brane.[3]

The typical energy of the bulk signals at emission is the temperature of the radiation on the brane, so at the moment of re-entry the signal must have an energy equal to the new temperature of the radiation. The Doppler effect only increases the redshift of the bulk signal. The energy of the signal re-entering the brane being smaller than the typical energy of a CMB photon, we will need large numbers of bulk signals, comparable to the number of photons in the CMB, in order to homogenize the universe. In such a situation a dangerously large amount of energy will leak into the bulk, possibly changing the result of nucleosynthesis. We therefore expect that the only contribution of the bulk signals is a deformation of the black-body spectrum of the Cosmic Microwave Background.

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[3]We would expect this result if we use the holographic description for the brane-world. The bulk signals are seen as excitations of some brane CFT, and the energy of the excitations red-shifts the same way as radiation on the brane.

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A. Static brane solutions

We now look for solutions of the equations $H = 0$ and $\dot{H} = 0$. We use the general solution for H found in ref. [7] (See also ref. [8]). The relevant equation is:

$$H^2 = \frac{\lambda^2}{4} - \frac{1}{2}\left(\frac{1}{l_-^2} + \frac{1}{l_+^2}\right) + \frac{1}{4\lambda^2}\left(\frac{1}{l_-^2} - \frac{1}{l_+^2}\right)^2 + \frac{k}{r^2} + \\ + \frac{1}{r^4}\left\{\frac{\mu_- + \mu_+}{2} - \frac{\mu_+ - \mu_-}{2\lambda^2}\left(\frac{1}{l_-^2} - \frac{1}{l_+^2}\right)\right\} + \frac{1}{r^8}\frac{(\mu_+ - \mu_-)^2}{4\lambda^2}. \tag{A.1}$$

For the symmetric case $l_+ = l_-$, $\mu_+ = \mu_-$, the equation simplifies to:

$$H^2 = \frac{\lambda^2}{4} - \frac{1}{l^2} + \frac{k}{r^2} + \frac{\mu}{r^4}. \tag{A.2}$$

In this case the conditions $H = 0$ and $\dot{H} = 0$ will give the constraints:

$$r_b^2 = -\frac{k}{2\mu} \qquad \text{and} \qquad \frac{\lambda^2}{4} - \frac{1}{l^2} - \frac{k^2}{4\mu} = 0. \tag{A.3}$$

Since we want to avoid the presence of naked singularities in the bulk, we consider only solutions with $\mu > 0$, which will restrict us to $k < 0$. In order to avoid this constraint we will allow for $\mu_+ \neq \mu_-$, while keeping $l_+ = l_-$ for calculational convenience. In this case there will be two "superluminal" bulk signals, one through each AdSS space, arriving at

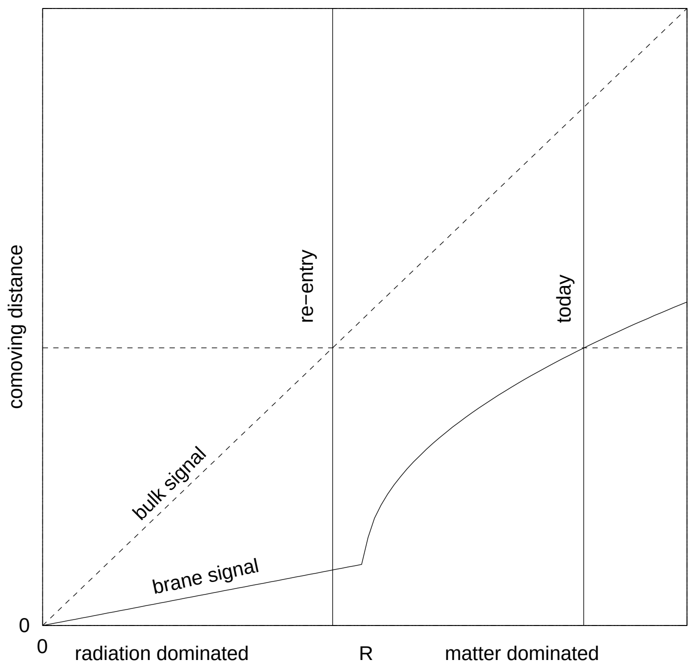

Figure 5: Comoving distance traveled by the bulk and the brane null signals as a function of the scale factor of the universe (redshift). We see that at the moment of re-entry the bulk signal has traveled a much larger comoving distance compared with the brane signal.

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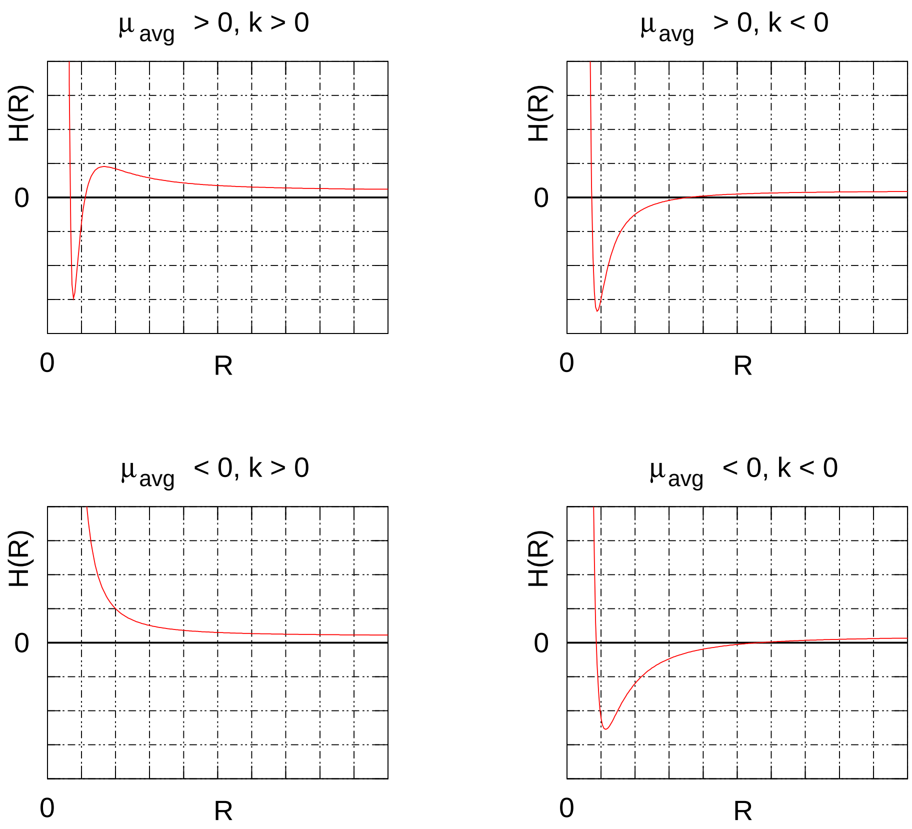

Figure 6: The dependence of the Hubble constant on the scale factor (redshift) for different signs of the curvature and black hole mass

the brane at different times since $\mu_+ \neq \mu_-$. The dependence of the Hubble constant on the scale factor of the visible universe:

$$H^2 = \frac{\lambda^2}{4} - \frac{1}{l^2} + \frac{k}{r^2} + \frac{\mu_+ + \mu_-}{2r^4} + \frac{1}{r^8}\frac{(\mu_+ - \mu_-)^2}{4\lambda^2} \tag{A.4}$$

is given in figure 5. We see that with the exception of the case $\mu_{\rm avg} = (\mu_+ + \mu_-)/2 < 0$, $k > 0$, when there is no extremum for $H(r)$, we can adjust the parameters λ, k, l, μ_+,μ_-, so that the extremum occurs for $H(r) = 0$.

B. Non-returning geodesics

We saw that geodesics for which $F(r_{\rm min}) = 0$ and $\frac{dF}{dr}|_{r=r_{\rm min}} = 0$ will not return to the brane. We show here that this condition is satisfied only for a particular geodesic: the condition $\frac{dF}{dr}|_{r=r_{\rm min}} = 0$ fixes the value of $r_{\rm min}$ as a function of the curvature and bulk black hole mass:

$$r^2_{\rm min} = \frac{2\mu}{k}\,. \tag{B.1}$$

Substituting this result in the equation $F(r_{\min}) = 0$, we obtain:

$$F(r_{\min}) = 1 - \frac{P^2}{E^2}\left(\frac{1}{l^2} - \frac{k^2}{4\mu}\right) = 0 \tag{B.2}$$

which fixes the value of the parameter P^2/E^2 characterizing the geodesic. We see that this condition selects a single geodesic, and in general this condition is not satisfied. Consequently, most of the geodesics will return to the brane.

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